ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

EXCITATION OF ACOUSTIC MODES IN FILM BOILING

B. M. Dorofeev

UDC 536.423.1:534.142

A photomultiplier has been used [1, 2] in a direct optical method of examining the fluctuations in the size of the vapor cavity around a heater; the output signals from the photomultiplier and hydrophone pass to a double-beam oscilloscope. The behavior of the vapor bubbles has also been examined by high-speed photography, with the film recording the waves. The data have been examined in detail for various resonance effects. Phase relationships between the oscillations and the acoustic signals imply that a cavity performs forced periodic pulsations under alternating sound pressure. The films and the phase shift between the oscillations and the acoustic pressure indicate that the periodic component in the noise is produced by the bubbles before detachment from the vapor cavity (see Fig. 1) and also when bubbles in the cavity fuse.

The volume fluctuations occur in these bubbles as follows. Film boiling produces large bubbles that are surrounded on all sides apart from the wire by cold liquid ($t_l < t_{sat}$); the rate v_- of vapor condensation in a bubble is governed by the surface area, so the rate increases with the size. If the rate v_+ of entry of vapor into the bubble from the cavity is greater than v_-, the bubble grows, but v_+ tends to fall rapidly immediately before the bubble becomes detached on account of the narrowing of the connecting neck. When v_- and v_+ become equal, the bubble still continues to grow for a short period on account of the inertia, and thus the bubble is not thermally in equilibrium ($v_+ < v_-$) when it attains its maximal size nor is it mechanically in equilibrium (the vapor pressure p_b in the bubble is less than the pressure in the liquid p_∞). Bubbles of maximum size are also not in equilibrium in bubble boiling, as has been found by direct experiment [3]. The result is that the bubble contracts, which draws the liquid in, and the liquid continues to move under its inertia and compress the bubble until v_- and v_+ become equal again. Therefore, the bubble at its minimum volume is also not in equilibrium ($v_+ > v_-$, $p_b > p_\infty$), so it begins to grow again. The bubble thus shows volume fluctuations due to the nonequilibrium thermal processes ($v_+ \neq v_-$) and the motion of the liquid under inertia. Several complete oscillation cycles can occur before the bubble becomes detached from the vapor cavity, and these produce a quasiperiodic pressure pulse of the corresponding length.

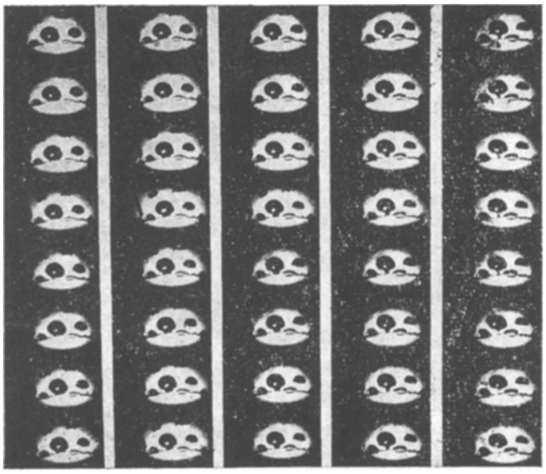

Fig. 1. Vapor bubble exciting a quasi-periodic sound pulse before detachment from the cavity. Investigated liquid — ethanol of core temperature 23 ±0.5°C; heater — a tungsten wire of length 25 mm and diameter 0.5 mm. Heat flux is 245 ± 10 W/cm^2. The sound marks (on the perforated film track) indicate the compression phase in the acoustic oscillations.

* All-Union Institute of Scientific and Technical Information.

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Original article submitted February 25, 1976.

STOCHASTIC MODEL OF CONTACT CONDUCTION IN A FILLED POLYMER WITH LOOSE PARTICLE PACKING

V. S. Novopavlovskii

UDC 536.212

There are numerous engineering applications of polymers filled with powders, in which the marked difference in conductivity in the components means that the contacts between the filler particles play a large part in the conduction. If the particles are very closely packed, which corresponds to a volume content p_{max}, all the particles are in contact, and the generalized conductivity of such a structure σ_{max} may be calculated from published formulas. The present study presents a method of defining the relative conductivity σ/σ_{max} in terms of the relative content p/p_{max} for loose packing, and it incorporates the random mode of contact between particles.

A structural model is proposed that allows one to calculate the mean number of contacts between particles; the particles are assumed to have a random distribution within a layer perpendicular to the flux and also it is assumed that there are contacts between particles in one layer (transverse contacts) and between adjacent layers (longitudinal contacts). The formulas for the number of contacts contain two geometrical parameters whose values may be defined approximately from theoretical calculation and then corrected by measurement of the electrical conductivity, e.g., for a plastic impregnated with graphite. The contact conductivity of a specimen having a relative length s in the conduction direction (the length scale is the mean particle size) is defined by the sum of the conductivities of the continuous contact chains between the boundary surfaces.

Figure 1 shows computer calculations, where the dashed line represents the curve given by Dul'nev's formula for a structure with interpenetrating components, which envisages that the material contains a continuous framework of contacting particles.

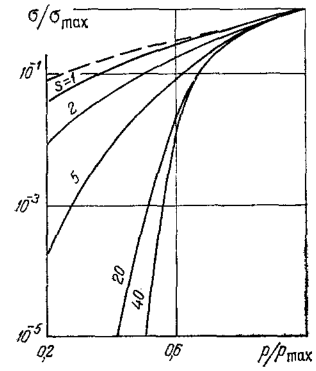

Fig. 1. Relative contact conductivity as a function of proportion of filler and specimen length.

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OBSERVED TEMPERATURE DEPENDENCE OF THERMOPHYSICAL CHARACTERISTICS FOR LAYERED VACUUM INSULATION

N. P. Pershin, R. S. Mikhal'chenko, and E. I. Shchirov

UDC 536.21

Layered evacuated insulation is widely used in efficient insulation of cryogenic devices; an apparatus and method are described for determining the thermal conductivity and specific heat of flat specimens of such insulation working over narrow temperature ranges (10-20°), which means that the temperature dependence of the thermophysical parameters can be determined. Results are reported on the thermal conductivity, thermal diffusivity, and specific heat for polyethylene terephthalate (PETP) film aluminized on both sides (Table 1) for the range from 50 to 350°K and a pressure in the insulation space of $3 \cdot 10^{-4}$ N/m^2.

Stationary and nonstationary techniques were used in measuring the thermophysical parameters; the error of measurement was evaluated with particular emphasis on the errors arising from the uneven distribution of the heat flux in the specimen.

The data on the thermal conductivity and specific heat may be represented as $A_0 + A_1T + A_2T^2 + A_3T^3$; Table 2 gives the values of the coefficients for $\lambda = f(T)$.

The empirical relationship for types II and III takes the form

$$c = -0.06 + 0.65 \cdot 10^{-2} T - 0.136 \cdot 10^{-4} T^2 + 1.4 \cdot 10^{-8} T^3 \text{ (kJ/kg·deg.)}$$

The effects of screen punching on the effective degree of blackness and thermal conductivity were also examined; results were recorded on $\lambda = f(T)$ and $a = f(T)$ for pressures in the insulated space for $3 \cdot 10^{-4}$ to $1.33 \cdot 10^{-1}$ N/m^2, and an equation was derived for $\lambda = f(T, P)$, which was compared with the measurements.

Results allow one to calculate the insulation behavior of such a system under a great variety of working conditions.

TABLE 1. Characteristics of Insulation Specimens

Type	Screen material	Packing material	Packing density, layers·cm^{-1}
I	PETP, crumpled, 10 μ	Glass cloth made of microfiber without bonding agent	22
II	PETP, corrugated, 8 μ	ÉVTI-7 glass cloth	16
III	PETP, corrugated, 8 μ, punched 2 × 10 mm	ÉVTI-7 glass cloth	17

TABLE 2. Coefficients and Polynomials for $\lambda = f(T)$. Vacuum $3 \cdot 10^{-4}$ N/m^2

Type	A_0	A_1	A_2	A_3
I	0	$0,154 \cdot 10^{-2}$	$-0,069 \cdot 10^{-4}$	$0,0409 \cdot 10^{-6}$
II	0	$-0,035 \cdot 10^{-2}$	$0,122 \cdot 10^{-4}$	$0,0074 \cdot 10^{-6}$
III	0,004	$0,280 \cdot 10^{-2}$	$-0,12 \cdot 10^{-4}$	$0,062 \cdot 10^{-6}$

Dep. 3323-76, August 9, 1976.
Original article submitted April 5, 1976.

MATHEMATICAL MODELING OF THE INTERACTION OF A STREAM OF VISCOUS INCOMPRESSIBLE LIQUID WITH A ROUGH WALL

L. V. Poluyanov and R. I. Ayukaev

UDC 531.15:539.371

One of the effective methods of intensification of processes of heat and mass exchange between a liquid stream and a solid surface is the regulation of the microgeometry of the latter. For example, the milling of projections or the mounting of diaphragms on the surfaces of heat-exchange pipes allows one to increase the heat exchange (when $Re \gg 1$) by 1.5-2.5 times [1, 2]; the use of grains with a developed microgeometry in granular filters in the separation of low-concentration suspensions increases the yield (when $Re \sim 1$) of the installations by 1.5-2.5 times [3]. The clarification of the dependence of the structure of the stream on the microgeometry of the surface over which it flows is of interest in this connection. Experimental methods prevail in the solution of such problems because of difficulties of a mathematical nature. In the report being published an attempt is made to study this problem analytically with the aim of obtaining some generalized estimates.

The model study of the effect of the microgeometry of the surface on the structure of a stream of incompressible viscous liquid flowing over it was conducted on the basis of the solution of the problem of Couette flow with a stationary corrugated wall. A generalized parameter — the wave number — is taken as the characteristic of the microgeometry by analogy with [4]. The flow is understood as plane and steady while the corrugation is considered as a small disturbance of the surface over which the flow occurs. The principal conditions of correspondence between real flow and the model under consideration are laminar attached flow over the surface and smallness of the amplitude of the disturbance in comparison with the distance between the walls. The solution is given for the limiting modes of flow of $Re \ll 1$ and $Re \gg 1$ and some generalizations are made to modes of flow with the joint manifestation of viscous and inertial effects (the most inconvenient for analytical studies).

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JET MODEL OF FLUIDIZATION AND ITS ENGINEERING APPLICATION

N. A. Shakhova*

UDC 532.517.4

The jet model, proposed by the author in 1965 on the basis of a theoretical and experimental study of turbulent gas jets discharging into a fluidized bed, treats the motion of the fluidizing agent from a gas-distribution grid of any construction in the form of jets which develop as in a homogeneous medium of different properties with partial or total degeneration of the jet flow into filtration flow. With the discharge of a fluidizing agent with a density less than the bed density the jet flow through the bed degenerates into bubble motion. Because of the disruption of the continuity of flow the pressure in the jet tongues increases, and when the separation

* Deceased.

pressure is reached, when the separation force exceeds the force of resistance, the jet breaks up and tongue–bubbles tear off.

The studies carried out over the last decade by the author and colleagues on the hydrodynamics and heat exchange in jets and in a fluidized bed made it possible to supplement and develop the theoretical foundations of the jet model of fluidization and its engineering application.

On the basis of the jet model one can explain the origin of the bubbles and the pressure pulsation in a fluidized bed, the nature of heat and mass transfer in a fluidized bed, and the effect of the bed height and the velocity of discharge from the openings of the gas-distribution grids on the amplitude and frequency of the pulsations and the intensity of heat and mass exchange.

The jet model allows one to make a number of recommendations:

1) to design and construct gas-distribution grids in intimate connection with the processes of heat and mass transfer;

2) to calculate the processes of heat and mass transfer separately in three established zones: the tongue zone, the bubble development zone, and the main volume of the bed;

3) to decrease the amplitude of the pressure pulsations, the dynamic entrainment, and the oscillations in bed height through the creation of a tongue zone only in the fluidized bed or through artificially introduced powerful gas jets;

4) to choose the blowers and conduct the strength calculations of apparatus containing a nonuniform fluidized bed with allowance for the pressure pulsations which develop and the rocking of the apparatus which results from them.

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Original article submitted January 22, 1976.

ANALYTICAL SOLUTION OF THE PROBLEM OF THE ACCURACY IN RECORDING THE TEMPERATURE PROFILE OF A HEATED LAYER OF CONDENSED SUBSTANCE WITH A FLAT PROBE

Yu. V. Chernov

UDC 536.5

When the temperature field of a heated layer of condensed substance is measured with a flat probe it is distorted because of the difference between the thermophysical characteristics of the probe and the material being studied. This leads to a methodological measurement error which can prove to be very considerable in a region of large gradients of the original temperature profile.

The methodological error is studied on the assumption that the rate of motion of the boundary is constant and the thermophysical characteristics do not depend on the temperature. These assumptions reflect the case, which is of principal practical interest, when the disturbing effect of the probe is small and the temperature dependence of the thermophysical characteristics can be allowed for in the solution obtained by calculating for the expected extreme values of the characteristics.

With $\lambda_z \gg \lambda$ the following calculating equations are obtained for the limiting values of the relative temperature measured by the probe, which are realized with a constant temperature at the moving boundary equal to its initial value (when the probe lies at a large distance from it) and a constant heat flux density at the moving boundary also equal to its value at the initial time:

$$\overline{T}_{\min} = \frac{\exp \frac{u}{a} \zeta}{\operatorname{sh} \frac{u}{a} \zeta + \operatorname{ch} \frac{u}{a} \zeta + \varkappa \operatorname{sh} \frac{u}{a} \zeta},$$

$$\bar{T}_{max} = \frac{\exp \frac{u}{a}\zeta}{\mathrm{sh}\frac{u}{a}\zeta + \mathrm{ch}\frac{u}{a}\zeta + \varkappa \mathrm{ch}\frac{u}{a}\zeta},$$

where u is the velocity; $a = \lambda/c\gamma$ is the coefficient of thermal diffusivity of the material under study; ζ is the distance from the front surface of the probe to the moving boundary; $\varkappa = (u/\lambda)c_z\gamma_z\delta_z$ is a dimensionless parameter of the probe; and c_z, γ_z, and δ_z are the heat capacity, density, and thickness of the probe. Equations for $\bar{T}_{min}$ and $\bar{T}_{max}$ with finite coefficients of thermal conductivity and equations for the temperature field are also obtained in the article.

Graphs of $\bar{T} = f[(u/a)\zeta]$ for several values of the probe parameters $\varkappa$ are presented for the case of $\lambda_z \gg \lambda$ which make it possible to determine the probe thickness on the basis of the allowable measurement error. The graphs can be useful for estimating calculations and in the case when the law of motion of the moving boundary as a function of the surface temperature is unknown.

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Original article submitted October 20, 1975.

EXPERIMENTAL STUDY OF CONVECTIVE HEAT EXCHANGE IN WINDOWS WITH SUN PROTECTION

E. S. Gul'karov and E. T. Artykpaev

UDC 536.24

Processes of convective heat exchange in window structures with two variants of sun protection — louver elements in the space between panes and outside in front of the glazing — are studied on a laboratory installation using an interferometer.

The thermal and air regimes in the model of the light opening were imitated by contact electrical heating of the elements of the opening and the use of a wind tunnel. Visual observation and photography were conducted over the entire cross section of the model of the light opening with subsequent combining of the interferograms obtained to estimate the overall pattern of the temperature field in the working zone and the hydrodynamics of the process. The relationship between the geometrical, hydrodynamic, and other parameters determining the intensity of heat exchange was established on the basis of an analysis of the experimental data. As a result, generalized functions are obtained (Table 1) for calculations of the average dimensionless coefficients of heat exchange at the surface elements of a light opening with several thermal-air regimes characteristic for the conditions of exploitation of transparent barriers in regions with a dry hot climate. The empirical coefficients c_1, c_2, and c_3 entering into the equations are represented in the form of graphic functions (Fig. 1) of the factors determining the heat-exchange process (the regime and the geometry of the system).

TABLE 1. Experimental Conditions and Results of Studies of Convective Heat Exchange in Window Openings Covered with Sun Protection

Arrangement of sun-protection louvers in profile of light opening	Regime in model of light opening	Heat-exchange surface in test region	Criterial dependence
In space between panes	Closed interlayer	Glass	$\overline{Nu} = 0.183\,(Gr \times Pr)^{1/3}$
		Louvers	$\overline{Nu} = (0.29 + 0.32p) \times (Gr \times Pr)^{0.29}$
	Ventilated interlayer	Glass	$\overline{Nu} = 6.07 \times Re^{0.30}$
		Louvers	$\overline{Nu} = c_1\, Re_{nom}^{0.76}$
Outside in front of glazing	Blowing by air stream (wind pressure)	Glass	$\overline{Nu} = c_2\, Re^{0.60}$
		Louvers	$\overline{Nu} = c_3\, Re_{nom}^{1/2}$

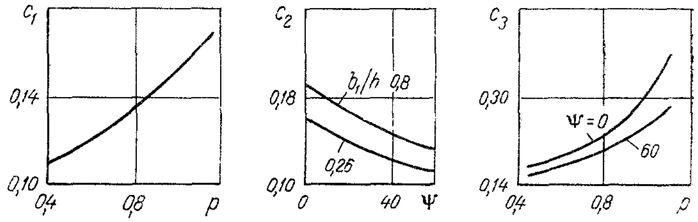

Fig. 1. Empirical coefficients c_1, c_2, and c_3 as functions of p, Ψ, and b_1/h.

By the experiments with an airtight interlayer it was established that the process of heat exchange at the glass surfaces intensifies with an increase in p with the other conditions being equal. Variation in the simplex p does not have an important effect on convective heat exchange during air filtration in the window space.

A departure from frontal blowing on the model of an opening with outside louvers leads to a decrease in the intensity of heat transfer from the surfaces of the elements of the light opening.

NOTATION

b, louver width, m; h, distance between louvers, m; φ, angle of inclination of louver to horizontal, deg; $p = b \cos \varphi / h$, characteristic of geometry of sun protection; c and n, empirical coefficients; $\overline{Nu}$, averaged Nusselt number; Gr, Grashof number; Pr, Prandtl number; $Re = Gl/\nu\gamma F$, Reynolds number; G, air flow rate in space between panes, kg/sec; l, width of air interlayer, m; ν, kinematic viscosity of air, m^2/sec; γ, air density, kg/m^3; F, cross-sectional area of air interlayer, m^2; Re_{nom}, nominal Reynolds number; Ψ, angle of attack, deg.

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Original article submitted April 18, 1975.

COMPARATIVE ANALYSIS OF CALCULATING FUNCTIONS FOR THE PROCESS OF COOLING OF CHANNELS WITH A GAS STREAM

V. G. Pron'ko, G. M. Leonova, and A. I. Raigorodskii

UDC 536.244

The analytical solutions of Anzelius, Schumann, and Nusselt for the process of cooling of long channels with gas are analyzed. The application of these solutions to the conditions of problems of cooling cryogenic equipment without further transformations is inadvisable for the following reasons. In the analytical solutions the heat capacity of the specimen material was taken as constant, whereas in cooling by cryogenic coolants the heat capacity varies quite considerably. The equations describing the temperature variation of the object being cooled must be solved jointly with the complicated equations describing the connection between the most important parameters of the refrigerating installations: primarily the coolant flow rate, its temperature at the inlet to the object being cooled, etc. In this connection it becomes necessary to simplify the form of the equations describing the temperature variation of the object being cooled.

In the article the analytical solutions are compared with experimental results on the cooling of channels by gaseous nitrogen and helium. It was shown by an analysis of the classical solutions and by the experimental results that when $St(l/d) \gg 1$ and $Bi \ll 1$ the intensity of heat transfer does not affect the nature of the variation in the temperatures of the object and the coolant, the only complex independent variables being the ratios of heat capacities of the body and the gas and the homochronicity number. Exponential functions obtained by the author, which are simple in form and which approximate the experimental data with an accuracy of ±15%, are recommended in the article. It is shown that results calculated by the Schumann solution using an average value of the heat capacity correspond considerably worse with the experimental data than results calculated

from exponential functions using values of the heat capacity corresponding to different sections of the temperature variation.

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Original article submitted December 9, 1975.

COMPUTER CALCULATION OF RADIANT HEAT EXCHANGE IN AN AXISYMMETRIC SYSTEM OF TWO BODIES ON THE EXAMPLE OF A CAVITY BOUNDED BY THE SURFACES OF A CONE AND A CYLINDER

S. P. Rusin

UDC 536.3

The problem is analyzed in the same formulation as in [1]. In contrast to [1, 2], both bodies can have a nonplane shape. The effective emissivity $\varepsilon_{ef}(\xi)$ of the cavity was sought (Fig. 1; the geometrical dimensions are normalized to the radius of the cavity opening). The values of $\varepsilon_{ef}(\xi)$ were determined from the system of two integral equations (1a), (1b) by the method of iterations (the notation here is analogous to that of [1]):

$$\varepsilon_{ef}(\eta_0) = (\varepsilon(\eta_0) + R(\eta_0)\Big(\int_0^{\eta_L} (\varepsilon_{ef}(\eta) f_1(\eta) - \varepsilon_{ef}(\eta_0)) K_1(\eta_0, \eta)\, d\eta + \int_0^{\xi_L} (\varepsilon_{ef}(\xi) f_2(\xi) - \varepsilon_{ef}(\xi_L) f_2(\xi_L)) K_2(\eta_0, \xi)\, d\xi + \varepsilon_{ef}(\xi_L) f_2(\xi_L) \varphi_2(\eta_0, F_2)\Big)) / (1 - R(\eta_0)\varphi_1(\eta_0, F_1)); \tag{1a}$$

$$\varepsilon_{ef}(\xi_0) = (\varepsilon(\xi_0) + R(\xi_0) \times \Big(\int_0^{\xi_L} (\varepsilon_{ef}(\xi) f_3(\xi) - \varepsilon_{ef}(\xi_0)) K_3(\xi_0, \xi) \times d\xi + \int_0^{\eta_L} (\varepsilon_{ef}(\eta) f_4(\eta) - \varepsilon_{ef}(\eta=0) f_4(\eta=0)) K_4(\xi_0, \eta)\, d\eta + \varepsilon_{ef}(\eta=0) f_4(\eta=0) \varphi_4(\xi_0, F_1)\Big)) / (1 - R(\xi_0)\varphi_3(\xi_0, F_2)); \tag{1b}$$

$$f_1(\eta) = E_0(\eta)/E_0(\eta_0); \quad f_2(\xi) = E_0(\xi)/E_0(\eta_0); \quad f_3(\xi) = E_0(\xi)/E_0(\xi_0); \quad f_4(\eta) = E_0(\eta)/E_0(\xi_0).$$

Some of the data for $\alpha = 0$ are summarized in Table 1. Values of q_Σ are presented there also (q_Σ is the ratio of the heat losses from the given cavity (gray model) to the heat losses from an absolutely black body at the

TABLE 1. Dependence of ε_{ef} and q_Σ on ε, η_L, and θ

ε	η_L	ε_{ef} for $\xi = 0$ and $\xi = \xi_L$			q_Σ		
		$\theta°$			$\theta°$		
		45	60	80	30	45	60
0,5	8	0,9913—0,9900	0,9897—0,9889	0,9885—0,9884	0,8425	0,8353	0,8353
0,75	8	0,9972—0,9965	0,9964—0,9960	0,9958—0,9957	0,9309	0,9319	0,9319
0,9	8	0,9990—0,9987	0,9988—0,9985	0,9985—0,9984	0,9752	0,9752	0,9752
0,5	4	0,9685—0,9553	0,9587—0,9529	0,9499—0,9520	0,8351	0,8343	0,8330
0,75	4	0,9905—0,9851	0,9868—0,9837	0,9831—0,9834	0,9308	0,9311	0,9309
0,9	4	0,9969—0,9950	0,9956—0,9944	0,9942—0,9942	0,9750	0,9749	0,9749
0,5	2	0,9204—0,8727	0,8878—0,8683	0,8549—0,8719	0,8295	0,8151	0,8114
0,75	2	0,9750—0,9539	0,9620—0,9515	0,9464—0,9528	0,9272	0,9253	0,9240
0,9	2	0,9920—0,9841	0,9874—0,9831	0,9814—0,9834	0,9737	0,9731	0,9726
0,5	1	0,8610—0,7684	0,7942—0,7658	0,7224—0,7783	0,7847	0,7664	0,7538
0,75	1	0,9524—0,9068	0,9223—0,9053	0,8827—0,9117	0,9139	0,9063	0,9006
0,9	1	0,9842—0,9664	0,9730—0,9657	0,9567—0,9681	0,9691	0,9664	0,9642
0,5	0,5	0,8109—0,6878	0,7132—0,6893	0,6116—0,7116	0,7388	0,7042	0,6798
0,75	0,5	0,9305—0,8658	0,8822—0,8667	0,8206—0,8792	0,8933	0,8765	0,8634
0,9	0,5	0,9761—0,9502	0,9574—0,9504	0,9309—0,9555	0,9614	0,9550	0,9497
0,5	0	0,7388—0,5395	0,6062—0,5173	0,5115—0,5019	0,6470	0,5798	0,5326
0,75	0	0,8946—0,7785	0,8220—0,7628	0,7585—0,7514	0,8423	0,8917	0,7697
0,9	0	0,9622—0,9134	0,9327—0,9061	0,9041—0,9007	0,9383	0,9198	0,9041

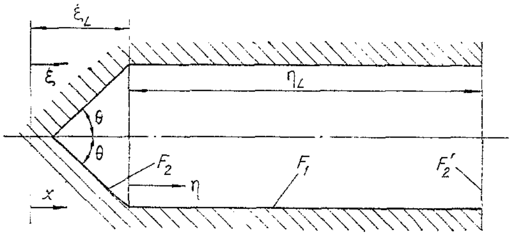

Fig. 1. Diagram of cavity.

same temperature). For θ = 90° our data agree with those of [3, 4]. The results of calculations for nonisothermal cavities (λ = 0.65 μ) are also presented. As a rule, the functions $\varepsilon_{ef}(\xi)$ have a minimum within $[0, \xi_L]$.

The numerical results obtained can be used in radiant pyrometry, in modeling porous materials by a system of cavities, and in creating artificial emitting surfaces, and the calculating algorithm itself can be used for calculations of thermal aggregates and structural elements having axial symmetry.

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Original article submitted May 5, 1975.

CALCULATION OF RADIANT HEAT EXCHANGE IN AN ISOTHERMAL SYSTEM OF SELECTIVELY EMITTING SOLID BODIES

S. P. Rusin

UDC 536.3

When the radiation wavelength λ, the field of temperatures T, and the field of the optical parameters ε and R are given the calculation of the radiant heat exchange is connected with the solution of an integral equation of the type (diffuse emission and reflection)

$$\varepsilon_{ef}(\lambda, M)\, E_0(\lambda, M) = \varepsilon(\lambda, M)\, E_0(\lambda, M) + R(\lambda, M) \int_{(F)} \varepsilon_{ef}(\lambda, N)\, E_0(\lambda, N)\, d\varphi(M, N), \quad M, N \in F \tag{1}$$

(the notation is the same as that in [1]).

Numerical integration over the spectrum is carried out after the calculation of $\varepsilon_{ef}(\lambda, M)$.

It is proposed to approximate the emission spectrum of solid bodies within a spectral band $\Delta\lambda$ by a linear dependence on λ (by a polynomial in the general case). The radiation characteristics calculated on a computer by the proposed method and in accordance with a "gray" model were compared for an isothermal cavity [1] of tungsten. The data coincided within limits of 5%. The agreement obtained is analyzed from the position of (1). Assuming that $\varepsilon_{ef}(\lambda)$ is a linear function of λ and using the theorem of the mean value of an integral, after integration of (1) in the interval $\Delta\lambda$ we have

$$\varepsilon_{ef}(\lambda_{av}, M) = (\varepsilon_\Sigma(M) + \int_{(F)} \varepsilon_{ef}(\lambda'_{av}, N)\, (T(N)/T(M))^4\, R(M, N)\, d\varphi(M, N))/\Delta F(M), \tag{2}$$

$$\lambda_{av} = \int_{(\Delta\lambda)} \lambda E_0(\lambda, M)\, d\lambda / \int_{(\Delta\lambda)} E_0(\lambda, M)\, d\lambda, \tag{2a}$$

$$\lambda'_{av} = \int_{(\Delta\lambda)} \lambda R(\lambda, M)\, E_0(\lambda, N)\, d\lambda / \int_{(\Delta\lambda)} R(\lambda, M)\, E_0(\lambda, N)\, d\lambda, \tag{2b}$$

$$\varepsilon_\Sigma(M) = \int_{(\Delta\lambda)} \varepsilon(\lambda, M) E_0(\lambda, M)\, d\lambda / (\sigma_0 T^4(M)), \tag{2c}$$

$$\Delta F(M) = \int_{(\Delta\lambda)} E_0(\lambda, M)\, d\lambda / (\sigma_0 T^4(M)), \tag{2d}$$

$$R(M, N) = \int_{(\Delta\lambda)} R(\lambda, M) E_0(\lambda, N)\, d\lambda / (\sigma_0 T^4(N)), \tag{2e}$$

and since in our case

$$\lambda_{av} \simeq \lambda'_{av}, \tag{3}$$

the system is quasimonochromatic. Moreover, the system is isothermal, and consequently $R(M, N) = R_\Sigma(M) = 1 - \varepsilon_\Sigma(M)$. When $\Delta F \to 1$ the quasimonochromatic system coincides with a "gray" model. It is just this which explains the closeness of the "gray" approximation to the real emission from a cavity in our case. For a non-isothermal system $R(M, N) \neq R_\Sigma(M)$ and one must use (2) for the calculations. Using the iteration method one can choose the quantity $\Delta\lambda$ such that Eq. (3) is satisfied with the required accuracy, i.e., the system is quasi-monochromatic with the required accuracy.

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THICKNESS OPTIMIZATION FOR THERMAL INSULATION ON A PIPELINE

M. Yunusov

UDC 519.9

If a pipeline is to carry materials at temperatures above or below the soil temperature, the thermal conduction in the soil may be accompanied by changes in phase state (for instance, thawing).

Any change in the physical state of a soil usually results in distortion of the pipeline, but this can be reduced or eliminated by appropriate insulation (provided this is adequate). Here we consider thickness optimization for such insulation for a pipeline carrying products at a temperature above that of the frozen soil.

The temperature distribution in the soil is determined by solving the Stefan's problem [1, 2], while the temperature of the product is derived from another formula [2, 3]. The optimality criterion is a functional dependent on the deviation of the radius of the phase-transition boundary in the soil from the permissible value, which itself is determined from strength considerations.

Physically, this means that the thawed depth must lie within a certain range subject to a safety margin on the amount of insulation and specified maximum and minimum thicknesses for the latter.

Numerical solution of a boundary-value problem of Stefan type is possible via an unconditionally stable scheme previously presented [1-3]. In this method, a nonlinear system of algebraic equations is derived, which is then solved by iterative fitting [4].

The optimum insulation thickness is determined numerically by successive approximations; it is proved that the iteration converges. The method is illustrated by calculating the optimum thickness for insulation on pipelines in frozen soils operating under various conditions, for which purpose a BÉSM-6 computer was used.

The results show that optimum thickness for glass-fiber insulation is 9.5-10.5 cm for the length up to 20 km, 7.5-8.5 cm for the length between 20 and 40 km, 6.5 cm for 40-50 km, 5 cm for 50-60 km, 3.5-4 cm for 60-80 km, and only 2.5 cm for the remainder. These results can be used in strength and stability calculations for pipelines.

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NONIDEAL THERMAL CONTACT BETWEEN PLATES UNDER NONSTATIONARY THERMAL CONDITIONS

Yu. M. Kolyano and O. F. Girnyak

UDC 539.377:537.47

The conditions for generalized nonideal thermal contact between plates via a thin intermediate layer are derived; the system contains no heat sources, but there is heat transfer to the environment at the sides in accordance with a generalized form of Newton's law. It is assumed that the temperatures and heating rates for the elements are initially zero.

The equations of thermal conduction for the intermediate layer are averaged over the width on the basis of linear variation of the temperature within the layer, and conditions are then formulated for generalized nonideal contact; if the system is symmetrical about the median plane, these conditions take the form

$$\Lambda_0 \frac{\partial^2 (T_1+T_2)}{\partial s^2} + 2\left[d_1\left(b_1\varphi_1+\frac{\partial T_1}{\partial n}\right) - d_2\left(b_2\varphi_2+\frac{\partial T_2}{\partial n}\right)\right] - 2l_0\left[A_0(T_1+T_2) - 2A_0\vartheta_e + \frac{C_0}{2}\frac{\partial (T_1+T_2)}{\partial \tau}\right] = 0,$$

$$\Lambda_0 \frac{\partial^2 (T_1-T_2)}{\partial s^2} + 6\left[d_1\left(b_1\varphi_1+\frac{\partial T_1}{\partial n}\right) + d_2\left(b_2\varphi_2+\frac{\partial T_2}{\partial n}\right)\right] - \frac{12}{R_h}(T_1-T_2) - 2l_0\left[A_0(T_1-T_2) - 2A_0\vartheta_e + \frac{C_0}{2}\frac{\partial (T_1-T_2)}{\partial \tau}\right] = 0,$$

where

$$\varphi_i(s,\ n,\ \tau) = \int_0^\tau \frac{\partial T_i(s,\ n,\ \zeta)}{\partial n} e^{\frac{\zeta-\tau}{\tau_r^{(i)}}} d\zeta;$$

$$b_i = \frac{\tau_r^{(i)} - \tau_r^{(0)}}{\tau_r^{(0)}\tau_r^{(i)}}; \quad d_i = \frac{\Lambda_i \tau_r^{(0)}}{\tau_r^{(i)}}; \quad (i=1,\ 2),$$

and $l_0 = 1 + \tau_r^{(0)}\partial/\partial\tau$, $\tau_r^{(i)}$ ($i = 0, 1, 2$) are the relaxation times for the heat fluxes in the plates and layer; $\Lambda_0 = 4\delta h\lambda_0$, $C_0 = 4\delta h c_v^{(0)}$, $A_0 = 2h\alpha_0$ are the reduced thermal conductivity, specific heat, and heat transfer from the surfaces $z = \pm\delta$, while $R_h = h/\lambda_0\delta$ is the thermal resistance of the layer; T_1, T_2, ϑ_e are the mean temperatures of the plates and environment; and τ is the time.

Particular cases of these conditions are discussed.

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CALCULATION OF THE TEMPERATURE IN A CLOSED VOLUME WITH DISTRIBUTED HEAT SINKS ON THE INNER SURFACE

R. K. Nikul'shin, E. F. Petriman, and A. V. Kontsev

UDC 536.2.01

A method is given for determining the nonstationary temperature of the air in a closed insulated volume having uniformly distributed heat sinks on the internal surface, which are provided by refrigeration or by a cryostat.

It is also assumed that the heat flux is one-dimensional and that Bi > 10, which results in boundary conditions of the first kind, with the temperature uniform over the internal surface of the chamber (the evaporator chamber), the temperature being that of the coolant t_0, while the air temperature t_{ch} in the chamber is constant, as are the heat-transfer factor α and the output P of the heat sources.

The air temperature within the chamber is defined from the heat-balance equation

$$\sum_i G_i c_i \frac{dt_{ch}(\tau)}{d\tau} = P - \alpha F [t_{ch}(\tau) - t_0(\tau)] \tag{1}$$

or

$$\frac{d\omega(\tau)}{d\tau} + b\omega(\tau) = \Phi(\tau), \tag{2}$$

where

$$b = \frac{\alpha F}{\sum_i G_i c_i}; \quad \Phi(\tau) = \frac{\alpha F \Theta_0(\tau) - P}{\sum_i G_i c_i}. \tag{3}$$

The evaporator temperature in the refrigerator varies exponentially:

$$\Theta_0(\tau) = \Theta_{0\max}(1 - e^{-m_0\tau}), \tag{4}$$

where m_0 is the cooling rate and $\Theta_{0\max}$ is the maximum temperature difference occurring when the machine has reached a steady state.

The solution to (2) for $\omega(\tau)_{\tau=0} = 0$ subject to (4) is

$$\omega(\tau) = \frac{1}{\sum_i G_i c_i}\left[\frac{\alpha F \Theta_{0\max} - P}{b}(1 - e^{-b\tau}) - \frac{\alpha F \Theta_{0\max}}{o - m_0}(e^{-m_0\tau} - e^{-b\tau})\right]. \tag{5}$$

Also, $\Theta_{0\max}$ is determined by solving the system for the steady state:

$$\begin{cases} Q_{re} = f(\Theta_0), \\ Q_{re} = \dfrac{\lambda_{in}}{\delta_{in}} F_{av}\Theta_{0\max} + P, \end{cases} \tag{6}$$

by specifying the time τ_c such that

$$\omega|_{\tau=\tau_c} = \omega_c, \tag{7}$$

and then (5) gives m_0.

The cooling source satisfies (7) if the following inequality is obeyed:

$$Q_{re}(\tau) \geqslant G_{in}c_{in}\frac{d\bar{u}(\tau)}{d\tau} + G_s c_s \frac{d\Theta_0(\tau)}{d\tau} + \sum_i G_i c_i \frac{d\omega(\tau)}{d\tau}, \tag{8}$$

where $\bar{u}(\tau)$ is the temperature of the insulation averaged over the volume:

$$\bar{u}(\tau) = \Theta_{0\max}\left[0.5(1 - e^{-m_0\tau}) + \frac{4m_0}{\pi^2}\sum_{k=1}^{\infty}\frac{(e^{-m_0\tau} - e^{-A_k\tau})}{(2k-1)^2(A_k - m_0)}\right]. \tag{9}$$

When $\Theta_0(\tau)$, $\omega(\tau)$, and $\bar{u}(\tau)$ have been determined, (8) is solved throughout the range $0 \le \omega(\tau) \le \omega_c$ in order to check the result for m_0.

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GENERAL METHODS OF DETERMINING CONTACT HEAT TRANSFER FOR A ROD SYSTEM

V. G. Pidodnya and V. M. Khorol'skii

UDC 536.21

Consider a system composed of n bounded rods $0 \le x \le a_i$ having a common contact region $x = 0$ (intermediate layer), in which specified amount of heat Q(t) is generated. The intermediate layer has a temperature T(t), and a liquid is pumped through this, which acquires the temperature of the contact region exactly.

The following system of equations describes the heat propagation in each rod on the basis of the heat transfer on the side faces:

$$\frac{\partial\theta_i}{\partial t} = \omega_i \frac{\partial^2\theta_i}{\partial x^2} - \omega_i\nu_i^2 (\theta_i - \delta_i) + \frac{\omega_i}{\lambda_i} \psi_i, \quad 0<x<a_i, \quad t>0. \tag{1}$$

The boundary conditions take the form

$$-\lambda_i \frac{\partial\theta_i}{\partial x}\Big|_{x=0} = q_i(t), \quad \lambda_i \frac{\partial\theta_i}{\partial x}\Big|_{x=a_i} = q_i^*(t), \quad \theta_i|_{t=0} = \varphi_i(x), \quad T|_{t=0} = T_0, \tag{2}$$

where $q_i(t)$ are unknown heat fluxes and $q_i^*(t)$ are known ones.

The temperature distributions in the contact areas satisfy the conditions for ideal thermal contact:

$$\theta_i|_{x=0} = T(t), \quad i=1, 2, \ldots, n. \tag{3}$$

It is also assumed that the following heat-balance equation applies for the contact zone:

$$Q(t) = \sum_{i=1}^{n} S_i q_i(t) + mT(t) + m_0 \frac{dT(t)}{dt}. \tag{4}$$

The theory of Green's functions is used to reduce the solution to (1) and (2) to that of a system of integral equations of Volterra type containing the weak singularity

$$\theta_i(x;\ t) = \frac{\omega_i}{\lambda_i} \int_0^t q_i(\tau)\, G_i(x;\ 0;\ t-\tau)\, d\tau + F_i(x;\ t), \tag{5}$$

where $F_i(x; t)$ are known functions and $G_i(x; \xi; t)$ are Green's functions that satisfy (1) and zero boundary conditions of the second kind.

An operational method is given for solving (4) and (5), which gives a general solution in terms of transforms.

An approximate method is also given, which involves time averaging; the heat transfer is considered for a specified time interval (0, t), while the heat fluxes $q_i(t)$ are averaged over this period. This averaging method results in a system of algebraic equations. Finally, a method is presented for solving (4) and (5) by approximating $q_i(t)$ in power-law form. This reduces the solution of (4) and (5) to that of algebraic equations of triangular type, which are simple to solve numerically.

NOTATION

ω_i, λ_i, h_i, thermal diffusivity, thermal conductivity, and heat-transfer coefficient; $\nu_i^2 = h_i P_i/\lambda_i S_i$, coefficients for heat transfer from the rod surfaces; S_i, P_i, cross-sectional area and perimeter of rod i;

m, thermal capacity of cooling liquid passed through contact space in unit time; $\psi_i = \psi_i(x; t)$, given functions describing the heat-source distribution in the rods; $\delta_i = \delta_i(x; t)$, temperatures of the external media.

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ANALYSIS OF THE SOLUTIONS OF SOME NONLINEAR PROBLEMS IN NONSTATIONARY HEAT CONDUCTION

N. M. Tsirel'man

UDC 536.21

By means of the substitution $\psi(\theta) = \int\limits_0^\theta f(\theta)\left\{\int\limits_0^\theta \left[\int\limits_0^\theta \frac{dy}{\varphi(\theta)}\right] dz\right\} dx$, the heat-conduction equation for an unbounded plate with a relative heat-conduction coefficient $f(\theta)$ and a volumetric heat capacity $\varphi(\theta)$ which depend on the temperature θ was reduced to the form

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2}{\partial\eta^2}\psi(\theta), \quad 0 < \eta < 1, \quad \tau > 0 \tag{1}$$

and was solved by a finite-difference method using an implicit absolutely convergent Crank—Nicholson scheme with the boundary conditions

$$\theta(0, \eta) = 0, \quad 0 \leqslant \eta \leqslant 1, \tag{2}$$

$$\theta(1, \tau) = 1, \quad \tau > 0, \tag{3}$$

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} = 0. \tag{4}$$

At the initial instant of time we used at the boundary of the region the conjunction function

$$\theta(1, \tau) = \begin{cases} 1 - \left(\dfrac{\tau - \tau_0}{\tau_0}\right)^2, & 0 \leqslant \tau \leqslant \tau_0, \\ 1, & \tau > \tau_0 \end{cases}$$

and the time step was taken to be $\tau_0 = 0.0001$. In addition to calculating $\theta(\eta, \tau)$, we set up a subprogram called "Interpretation" which carried out on each time layer a search for values of the temperature θ equal to 0.1, 0.2,..., 0.9 or the points closest to them in the points of the space $\eta = 0.05, 0.10, 0.15, \ldots, 0.95$.

The selected method of solution was checked with a control problem [1] on a temperature wave in an unbounded plate with $f(\theta) = 0.5\,\theta^2$ and $\varphi(\theta) = 1$, which has an analytic solution, and was then applied to the following cases:

$$\text{a) } f(\theta) = 1 + \alpha\theta; \quad \text{b) } f(\theta) = \exp(\alpha\theta); \quad \text{c) } f(\theta) = \frac{1}{(1-\alpha\theta)^2} \text{ for } \varphi(\theta) = 1.$$

The results obtained enable us to conclude that the numerical calculation of temperature fields in the peripheral parts of bounded bodies may be replaced with the solution of a nonlinear problem for an "identical" half-space (for boundary conditions of the first kind), when it is possible to reduce partial-differential-equation problems to ordinary-differential-equation problems whose solution can be obtained more simply, or even to pass in some cases to linear equations. In those special cases in which $1 \leqslant \int\limits_0^1 f(\theta)\,d\theta \leqslant 2$, the temperature field in the peripheral part of a bounded body for any monotonic $f(\theta)$ can be determined with sufficient accuracy by using, for example, the available analytic solutions of [2] for the half-space without satisfying the condition of "identity" of the bounded body and the half-space with respect to $f(\theta)$.

All the results cited are also valid for the case $\varphi(\theta) \neq 1$ if instead of θ we take

$$u = \int\limits_0^\theta \varphi(\theta)\,d\theta.$$

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SOLUTION OF HEAT-CONDUCTION PROBLEMS FOR A HOLLOW SOLID OF REVOLUTION WITH VARIABLE BOUNDARY CONDITIONS

Yu. L. Khrestovoi

UDC 536.201

The axially symmetric two-dimensional temperature field of a hollow bounded solid of revolution with a distributed heat source F and arbitrary initial distribution, with length L, outer diameter R, and inner diameter r_0, when the boundary conditions are nonsymmetric boundary conditions of the third kind and the temperatures of the medium are functions of time and the coordinates, is described by the system

$$\frac{\partial t}{\partial v}=\frac{\partial^2 t}{\partial z^2}+\frac{\varsigma^2}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)+F(z,r,v),\quad t=t(z,r,v), \tag{1}$$

$$\mu\leqslant r\leqslant 1,\quad 0<\mu<1,\quad 0\leqslant z\leqslant 1,\quad 0\leqslant v<\infty,$$

where

$$v=\frac{a\tau}{L^2},\quad z=\frac{\xi}{L},\quad r=\frac{\rho}{R},\quad \mu=\frac{r_0}{R},\quad \varsigma=\frac{L}{R}.$$

The boundary conditions are

$$\begin{aligned}&\left.\frac{\partial t}{\partial r}\right|_{r=\mu}=h_\mu\,[t|_{r=\mu}-t_{f_\mu}(z,v)],\quad \left.\frac{\partial t}{\partial r}\right|_{r=1}=-h\,[t|_{r=1}-t_f(z,v)],\\ &\left.\frac{\partial t}{\partial z}\right|_{z=0}=h_0\,[t|_{z=0}-t_{f_0}(r,v)],\quad \left.\frac{\partial t}{\partial z}\right|_{z=1}=-h_1\,[t|_{z=1}-t_{f_1}(r,v)],\end{aligned} \tag{2}$$

where

$$h_\mu=\frac{\alpha_\mu R}{\lambda},\quad h=\frac{\alpha R}{\lambda},\quad h_0=\frac{\alpha_0 L}{\lambda},\quad h_1=\frac{\alpha_1 L}{\lambda}.$$

The initial distribution is

$$t|_{v=0}=\varphi_0(z,r). \tag{3}$$

Here ξ and ρ are dimensional coordinates, λ and a are the thermal conductivity and thermal diffusivity, t_f and α are the temperatures of the medium and the heat-exchange coefficients, the indices refer to the heat-exchange surface.

By successive reduction of the boundary conditions to homogeneous boundary conditions and subsequent application of the Hankel transform

$$T_H=\int\limits_\mu^1 rTV_0(pr)\,dr,\quad V_0(pr)=cJ_0(pr)-dY_0(pr),$$

$$c=Y_1(p\mu)+\frac{h_\mu}{p}Y_0(p\mu),\quad d=\frac{h_\mu}{p}J_0(p\mu)+J_1(p\mu),\quad p=\{p_i,\ i=1,2,\ldots,\infty\}, \tag{4}$$

$$\left[\frac{h}{p}J_0(p)-J_1(p)\right]\left[\frac{h_\mu}{p}Y_0(p\mu)+Y_1(p\mu)\right]=\left[\frac{h}{p}Y_0(p)-Y_1(p)\right]\left[\frac{h_\mu}{p}J_0(p\mu)+J_1(p\mu)\right]$$

(J and Y are Bessel functions) and the Fourier transform

$$u_F = \int_0^1 u W_0(kz)\,dz, \quad W_0(kz) = \cos kz + \frac{h_0}{k}\sin kz,$$

$$k = \{k_i,\ i = 1, 2, 3, \ldots, \infty\}, \quad \operatorname{ctg} k = \frac{k^2 - h_0 h_1}{k(h_0 + h_1)} \tag{5}$$

the solution can be written as

$$t = t_c + \sum_k \sum_p \frac{V_0(pr) W_0(kz) \exp(-M_{cr} v)}{\int_\mu^1 r V_0^2(pr)\,dr \int_0^1 W_0^2(kz)\,dz} \left\{ \int_\mu^1 \int_0^1 (\varphi_0 - t_c|_{v=0}) \right.$$

$$\left. \times r V_0(pr) V_0(kz)\,dr dz + \int_0^v \int_\mu^1 \int_0^1 \left(\frac{\partial^2 t_c}{\partial z^2} - \frac{\partial t_c}{\partial v} + \frac{\varsigma^2}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial P_\mu}{\partial r}\right) - p^2 P\right) + F\right) r V_0(pr) W_0(kz) \exp(M_{cr} v)\,dr dz dv \right\}, \tag{6}$$

where

$$t_c = P + P_1, \quad M_{cr} = k^2 + \varsigma^2 p^2, \quad P = g_0 + \frac{h_1 z^2}{h_1 + 2}(g_0 + q_1),$$

$$P_\mu = t_{f\mu} + \frac{h}{(1-\mu)h + 2}(t_f - t_{f\mu}) \frac{(r-\mu)^2}{1-\mu}, \tag{7}$$

$$g_0 = t_{f_0} - P_\mu|_{z=0} + \frac{1}{h_0} \cdot \left.\frac{\partial P_\mu}{\partial z}\right|_{z=0}, \quad g_1 = t_{f_1} - P_\mu|_{z=1} - \frac{1}{h_1} \cdot \left.\frac{\partial P_\mu}{\partial z}\right|_{z=1}.$$

We also obtained a solution for a one-dimensional problem for a hollow disk, a special case of which is a distribution in an unbounded hollow cylinder.

The solution can easily be generalized to a continuous solid of revolution. As $\mu \to 0$, $P_\mu = t_f$,

$$W_0(pr) = J_0(pr), \quad p = hJ_0(p)/J_1(p).$$

The solutions given above can be used for estimating the accuracy of numerical, analog, and other approximate methods of solution of heat-conduction problems.

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ONE METHOD OF SOLVING PROBLEMS OF HEAT AND MASS EXCHANGE WITH STOCHASTICALLY PERTURBED COEFFICIENTS OF THE EQUATION

P. A. Moroz and A. P. Korostelev

UDC 518.519.2

The first boundary-value problem is analyzed for the equation

$$k(x, y)\left[\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2}\right] - c(x, y) \cdot u(x, y) + f(x, y) = 0,$$

where the coefficients k(x, y) and c(x, y) represent Gaussian random fields. Let $k(x, y) = k_0(x, y) + \sigma k(x, y|\omega)$, where $k_0(x, y)$ is the average value of the coefficient; σ is a small parameter; and $k(x, y|\omega)$ is an isotropic random field with a correlation function $B(\vec{r})$. Analogous assumptions are made relative to the function c(x, y). It is proposed to solve such problems by the Monte Carlo method [1], realizing the trajectories of diffusional processes. In contrast to the well-known "lottery" methods for the random fields k(x, y) and c(x, y), it is proposed to model the random properties of the medium by a time-dependent "noise" $\xi(t|\omega)$ in the realization of the diffusional trajectories. Thus, with $c(x, y) \equiv 0$ it is proposed to model the trajectories of diffusional processes determined by the stochastic differential equations $dX_t^0 = \sqrt{k_0(X_t^0)}dw_t$ and $dX_t = \sqrt{k_0(X_t) + \xi(t|\omega)}dw_t$ where w_t is a two-dimensional Wiener process [2]; then quantities (functionals of the trajectories X_t^0 and X_t of the processes) of the type

$$J_0 = \int_0^{\tau_0} f(X_t^0)\,dt; \quad J = \int_0^{\tau} f(X_t)\,dt$$

are calculated, where τ_0 and τ are the times of departure of the respective trajectories from the region in which the solution is sought.

On the assumption that the trajectories of the processes emerge from the point (x, y), it is shown that the average value of $(J_0 - J)^2$ is an estimate of the dispersion of the solution u(x, y) at this point.

An analogous estimate of the dispersion is demonstrated for the case of $c(x, y) \equiv 0$.

To obtain such estimates it is important to correctly "transfer" the correlation of the coefficient k(x, y) in space into the correlation of the random "noise" $\xi(t|\omega)$ in time. The appropriate equation is obtained for the determination of the correlation function $\xi(t|\omega)$ through $B(\vec{r})$.

The proposed method can be realized both on a specialized probability hybrid (analog-digital) calculating complex [3] and on a universal computer, although the realization is less efficient in the latter case.

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A LOCAL FORMULATION OF THE EQUATION OF EXERGETIC BALANCE

G. P. Yasnikov and V. S. Belousov

UDC 536.70

The substantial and local forms of the equation of exergetic balance, in the derivation of which methods of the thermodynamics of irreversible processes are used, are obtained for a stream in external conservative fields in the presence of chemical reactions. The substantial equation of exergetic balance in the Lagrangian form has the form

$$\rho \frac{de^{(L)}}{dt} = -\vec{\nabla}\cdot\left(\vec{J}_e^q + \sum_k e_k \vec{J}_k\right) + \dot{e}_r + \dot{e}_{diff} - T_0\sigma^* - \dot{l}_v'.$$

The physical meanings of the terms entering into this equation are as follows: $\vec{J}_e^q = \tau_e \vec{J}_q - \tau_e \sum_k h_k \vec{J}_k$ is the flux of heat exergy carried by heat conduction; $\vec{J}_q$ and $\vec{J}_k$ are the fluxes of heat and matter; τ_e is the exergetic temperature; $\tau_e \sum_k h_k \vec{J}_k$ is the flux of heat exergy from the mass of components; $\sum_k e_k \vec{J}_k$ is the diffusional exergy flux; $\dot{e}_r = -(\hat{P}^v : \vec{\nabla}\vec{V})\tau_e$ is the exergetic power of the viscous forces; $\hat{P}^v$ is the viscous stress tensor; $\vec{V}$ is the stream velocity; $\dot{e}_{diff} = \tau_e \sum_k \vec{J}_k \vec{F}_k$ is the exergetic power of the diffusional flux of potential energy; $T_0\sigma^* = T_0\left[-\vec{J}_q \times \frac{\vec{\nabla}T}{T^2} - \sum_k \vec{J}_k \cdot \vec{\nabla}\left(\frac{\mu_k}{T}\right) + \sum_j \frac{A_j J_j}{T}\right]$ is the power of the exergetic losses due to heat conduction, diffusion, and chemical reactions; T and T_0 are the temperatures of the stream and the medium; μ_k is the chemical potential; A_j is the affinity of the j-th chemical reaction; J_j is the chemical reaction rate; $\dot{l}_v' = \rho(dl'/dt)$ is the power corresponding to the external work $dl' = -vdp$.

The quantity $e^{(L)}$ represents the specific exergy of the stream (in the Lagrangian form) in the system of the center of mass of an isolated element. The total exergy (in the Eulerian form)

$$e^{(E)} = e^{(L)} + \frac{1}{2} v^2 + \varphi,$$

which takes into account the mechanical energy of the stream ($\frac{1}{2}v^2 + \varphi$), is introduced for the transition to the local form of the equation of exergy balance.

The equation obtained allows one to introduce the local flux of exergy and its sink (exergetic losses). The results obtained can be used for the analysis of continuous multicomponent systems in which chemical reactions and processes of diffusion and heat conduction occur and viscous friction exists, from the point of view of the possibility of the performance of external useful work by them.

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A GENERALIZED GIBBS EQUATION FOR NONAUTONOMOUS PHASE INTERFACE REGIONS

É. V. Veitsman

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A generalized Gibbs equation is obtained for a phase interface region (PIR) when the field of (quasi) external forces is nonsteady and its curl is equal to zero:

$$T d\overset{+}{s} = d\overset{+}{u} - \tau_{ij} d\varepsilon_{ij}/\rho - \sum_{k} \overset{+}{\mu}_k d\overset{+}{N}_k - \sum_{k} \overset{+}{N}_k d_t [(\tau_k)_{ij} \delta_{ij}/\rho]. \quad (1)$$

The linear thermodynamics of irreversible and nonsteady processes taking place in the PIR (a surface layer) is developed using (1). The method of I. Prigogine [1] is used in its development, i.e., the entropy production $\sigma[S]$, the dissipative function ψ, and the generalized fluxes and forces are sought. In the process (1) and the local energy balance are used. The local balance of potential energy is supplemented by a term allowing for the nonsteady nature of the (quasi) external forces. The theory of surface forces presented in [2, 3] is used. The phenomenological laws are formulated:

$$v_\Omega = \sum_{\Omega'=1}^{R} a_{\Omega\Omega'} A_{\Omega'} + a_{\Omega q} (X_q)_\beta \delta_\beta + \sum_{k=2}^{N} a_{\Omega k} (X_k^*)_\beta \delta_\beta + a_{\Omega p} X_{i\beta} \delta_{i\beta},$$

$$(I_k^*)_\beta = \sum_{\Omega=1}^{R} a_{k\Omega} A_\Omega \delta_\beta + a_{kq} (X_q)_\beta + \sum_{k=2}^{N} a_{kk'} (X_{k'}^*)_\beta + a_{kp} X_{i\beta} \delta_i,$$

$$(I_q)_\beta = \sum_{\Omega=1}^{R} a_{q\Omega} A_\Omega \delta_\beta + a_{qq} (X_q)_\beta + \sum_{k=2}^{N} a_{qk} (X_k^*)_\beta + a_{qp} X_{i\beta} \delta_i,$$

$$P_{i\beta} = \sum_{\Omega=1}^{R} a_{p\Omega} A_\Omega \delta_{i\beta} + a_{pq} (X_q)_i \delta_\beta + \sum_{k=2}^{N} a_{pk} (X_k^*)_i \delta_\beta - \eta \left(\frac{\partial w_i}{\partial x_\beta} + \frac{\partial w_\beta}{\partial x_i} \right) + \left(\frac{2}{3} \eta - \zeta \right) \delta_{i\beta} \frac{\partial w_j}{\partial x_j}; \quad i, j = 1, 2, 3.$$

The first attempt at the formulation of the phenomenological laws for anisotropic regions was presented in [4]. However, in [4] a number of inaccuracies were committed which are eliminated in the present article.

NOTATION

T, absolute temperature; $\overset{+}{s}$, $\overset{+}{u}$, $\overset{+}{\mu}_k$, specific entropy, internal energy (J/kg), and chemical potential of substance of type "k"; τ_{ij}, ε_{ij}, stress and deformation tensors; ρ, ρ_k, density of substance and partial density of substance of type "k"; $\overset{+}{N}_k = \rho_k/\rho$; $(\tau_k)_{ij}$, component of τ_{ij} due to the presence in the system of a quasi-external specific force acting on particles of type "k"; w_i, w_j, velocity vector of center of mass; δ_{ij}, $\delta_{i\beta}$, Kronecker object; x_β, coordinate; $P_{i\beta}$, I_q, $(I_k^*)_\beta$, generalized fluxes of processes of viscous flow, heat transfer, and mass transfer; $X_{i\beta}$, X_q, $X^k{}_\beta$, $(X_k^*)_\beta$, generalized forces corresponding to them; $a_{kk'}$, a_{pq}, etc., phenomenological coefficients; η, ζ, coefficients of viscosity and volumetric viscosity; δ_i, δ_β, unit dimensionless vectors with matrices (111).

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THERMAL PROCESSES IN CUTTING WITH SPECIFIED TOTAL HEAT-SOURCE OUTPUT

M. A. Aranzon, G. A. Kulakov, and V. M. Khorol'skii

UDC 621.791.011

Steady-state heat transfer during cutting is considered for a three-body system: cutter, component, and swarf. It is assumed that the heat-balance equation applies to the cutting zone:

$$Q = Q_1 + Q_2 + Q_3,$$

where $Q_i = q_i s_i$; $i = 1, 2, 3$ are the heat fluxes in the cutter, component, and swarf via contact areas s_i; $Q = p_z v$ is the total output of the heat sources; p_z is the principal component of the cutting force; v is cutting speed; $s_1 = bl$; $s_2 = bl_2$; $s_3 = bl_3$; $l_3 = l_f + l_r$; $l_2 = l_3 + l_d$; b is cut width; l_f and l_r are the lengths of the contact areas on the front and rear faces of the cutter; l_d is the length of the deformation area; and $l = l_f + l_r$.

The temperature T at the edge of the cutter (the contact temperature) is common to all the bodies in contact; the contact temperature for each body is specified as

$$T = Q_i \gamma_i; \; i = 1, 2, 3,$$

where γ_i are the thermal potentials, which satisfy the heat-conduction equations and certain initial and boundary conditions.

We substitute the expressions for the heat fluxes in terms of the common temperature T into the heat-balance equation to get a solution for the contact problem as

$$T = Q\left[\sum_{i=1}^{3} \frac{1}{\gamma_i}\right]^{-1}; \; Q_i = T\frac{1}{\gamma_i}; \; i = 1, 2, 3.$$

The main difficulty here is to calculate the γ_i, which are dependent on the speeds of the heat sources, the geometry of the bodies, the shapes of the contact areas, and other factors.

Heat-source methods have given expressions for the thermal potentials for various conditions; for instance, if the cutter is taken as an octant and the sources are distributed as quarter-circles of radii l_f and l_r, then the following relations apply for weak cooling and vigorous cooling, respectively:

$$\gamma_1 = 2/(blh); \; \gamma_1 = 1/(\lambda_1 b),$$

where λ_i and h are the thermal conductivity and heat-transfer coefficient.

If the component is represented as a half-space, with distributed heat sources moving over the surface, then

$$\gamma_2 = \frac{1}{2\lambda_2 b} F\left(\frac{vl_2}{4\omega_2}\right); \; F(\xi) = \sqrt{\frac{2}{\pi\xi}}.$$

The swarf is taken as a plate with distributed heat sources moving over the end. The heat released by the strain and friction on the front face of the cutter is incorporated to give

$$\gamma_3 = \left[ntS \frac{\lambda_3}{\omega_3} v + 2(1-n)\lambda_3 bf\left(\frac{vl_3}{4\omega_3 k_e}\right)\right]^{-1},$$

where $f(\xi) = [F(\xi)]^{-1}$; k_e is the shrinkage factor; n is a coefficient representing the heating of the swarf ($0 \le n \le 1$); and tS is the cross-sectional area of the cut.

As it is comparatively simple to determine the total amount of heat deposited in the cutting zone, this method provides a good means of comparing theoretical temperatures with observed ones. Such comparison has been performed in the cutting laboratory at Kuibyshev Polytechnical Institute, and it has been found that the discrepancy between the temperatures does not exceed 10%.

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THE ELASTOPLASTIC STATE IN A SOLIDIFIED CYLINDRICAL CASTING

V. A. Zhuravlev and A. L. Golovanov

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The state of strain in a casting has been examined with the body represented as an ideal elastoplastic body subject to Trask's yield condition. The crystallizing phase is considered as consisting of a plastic zone in which the relaxation time for the thermoelastic stresses is negligible by comparison with the characteristic solidification time, together with an elastic zone in which the stress relaxation time is very large. The yield point is taken as a linear function of temperature.

The temperature distribution in the crystallizing casting is derived from the corresponding axially symmetrical Stefan problem [1]:

$$T(r;\ \mathrm{Fo}) = \frac{1}{4\,\mathrm{Fo}^*}\exp\left(-\frac{1}{4\,\mathrm{Fo}^*}\right)\left[\overline{\mathrm{Ei}}\left(\frac{r^2}{4\,\mathrm{Fo}}\right) - \overline{\mathrm{Ei}}\left(\frac{1}{4\,\mathrm{Fo}^*}\right)\right].$$

Transfer to dynamic variables is used in handling the mechanical problem: the strain rates and the rates of change in the stress tensor. This approach has been used [2] in calculating thermoelastic stresses for a crystallizing cylindrical casting. The inverse transformation is performed in accordance with

$$\sigma = \int_\tau^t \dot{\sigma}(r;\ t)\,dt,$$

where τ is the moment of attachment of a point to the moving phase boundary and t is the current time. The conditions of the boundaries of the elastic and plastic zones are used to derive expressions for the components of the stress tensor in each zone together with the coordinates of the boundaries.

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CALCULATION OF THE TEMPERATURE DISTRIBUTION IN A SOLID SUBJECT TO NONLINEAR BOUNDARY CONDITIONS

Yu. V. Vidin

UDC 536.2.01

It is common practice to examine complex nonlinear heat-transfer processes by describing the transport in terms of integral equations in which the unknown function is the heat flux at the surface of the body [1]. Unfortunately, the solutions to such equations cannot be represented in closed analytical form, apart from rare special cases.

A method has been described [2, 3] for linearizing the boundary conditions by using the solution for a thin body as the kernel in the integral transform; then one uses the following transform [2]:

$$U - U_0 = N \int_{\vartheta_0}^{\vartheta} \frac{d\eta}{f(\eta)}, \tag{1}$$

to get a linear boundary condition of the second kind, while if one uses the following [3]:

$$\frac{U}{U_0} = \exp - N \int_{\vartheta_0}^{\vartheta} \frac{d\eta}{f(\eta)}, \; N = \text{const}, \tag{2}$$

one gets a boundary condition of the third kind. Here $f(\vartheta)$ is the specified heat flux, which is dependent on the surface temperature. When (1) or (2) is used in the modified-conduction equation, the nonlinear term can be eliminated for bodies of medium heat capacity. Therefore, the integration for the initial nonlinear case can be reduced to solution of a linear transport problem subject to boundary conditions of the second or third kind.

However, many numerical solutions are available for nonlinear cases and can be utilized in examining heat propagation when there are more complex nonlinearities in the boundary conditions. For this purpose one can use the following general transform:

$$\int_{U_0}^{U} \frac{d\beta}{g(\beta)} = \int_{\vartheta_0}^{\vartheta} \frac{d\eta}{f(\eta)}, \tag{3}$$

where the function g(U) describes the heat-transfer law for a standard problem for which a solution is available in analytical, graphical, or tabular form. It is clear that (1) and (2) are particular cases of (3).

It is best here to select a standard problem from those available such that g(U) corresponds best to the structure of $f(\vartheta)$.

The expression for the complex appearing in the modified conduction equation is as follows when (3) is used in the one-dimensional case:

$$P = \frac{f' - g'}{g} \left(\frac{\partial U}{\partial \psi} \right)^2 .$$

This means that P is small in magnitude in both instances; first, if the heat capacity of the component is not too large, we have $(\partial U/\partial \psi)^2 \to 0$, and, secondly, P tends also to zero when the derivatives f' and g' do not differ substantially. If g(U) is chosen appropriately, the second condition can be met and thus the effects of P can be minimized.

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TEMPERATURE DISTRIBUTION IN A SEMIINFINITE CYLINDRICAL SHELL

A. N. Kulik, L. S. Gul'chevskii, and Ya. I. Podkova

UDC 539.377

The temperature distribution in a semiinfinite cylindrical shell has been determined together with that in a thin annular plate attached to the end, with the outer edge of the plate welded to a cermet unit in an electron-beam device. The theoretical system consists of semiinfinite plates and a strip heated by a line source moving over the end at a constant speed.

Differential equations for thin plates with specified heat fluxes at the side surfaces are utilized along with expressions for the internal heat sources in order to determine the temperature distribution in the plate:

$$\Lambda p^2 T + w = -(q^+ + q^-),$$

$$\Lambda p^2 T^* - \frac{12}{r} T^* + w^* = -3(q^+ - q^-),$$

where

$$\Lambda = 2\lambda\delta;\ p^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{a}\frac{\partial}{\partial \tau};$$

$$T = \frac{1}{2\delta}\int_{-\delta}^{+\delta} t\,dz;\ T^* = \frac{3}{2\delta^2}\int_{-\delta}^{+\delta} zt\,dz$$

are the integral characteristics of the temperature t of the plate and

$$w = \int_{-\delta}^{+\delta} W\,dz;\ w^* = \frac{3}{\delta}\int_{-\delta}^{+\delta} zW\,dz,$$

where W is the density of the heat sources, $q^{\pm}$ are the specified heat fluxes at the side surfaces, and $r = 2\delta/\lambda$ is the internal thermal resistance of the plate.

The solution is derived as follows. The temperature distribution in the plate is determined for a line source moving over the end in the presence of heat sink on the side welded to the semiinfinite plate (cermet). Then the temperature distribution is determined for a semiinfinite plate heated on the end by a flux equal to that absorbed by the sink, which was not previously known.

As the mean temperatures must be equal at the interface between the annular plate and the semiinfinite plate, the unknown heat flux can be determined, and this serves to define the temperature distribution in each of the two bodies.

Calculations have been performed for the quasistationary state to give the dimensionless temperature at the interface between the two plates; a study has also been made of the effects of heat transfer from the surface of the semiinfinite plate.

It is found that the maximum temperatures at the joint between the two plates occur behind the heat source, where the temperature gradients are minimal.

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TRANSFER-FUNCTION DETERMINATION FOR A HEAT EXCHANGER HAVING A SPATIAL DETERMINANT OF ORDER n

I. G. Chumak, A. I. Kokhanskii,
and S. N. Rogovaya

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The transient state in a heat exchanger may be examined by means of differential equations in partial derivatives; there are various methods of solving these equations in terms of the spatial coordinates, but they all have the disadvantage that it is very complex or virtually impossible to derive the transcendental transfer functions for such a system when the order of the spatial determinant for the system is greater than two. A method is given for defining the transfer functions for a system whose equations have a spatial determinant of order n, for which purpose the system is put in operator form as follows subject to zero initial and boundary conditions (for a packed air cooler):

$$\begin{aligned} t_a(s,\ p) &= \frac{a_2}{s+A_1(p)}\, t_w(s,\ p) - \frac{a_3}{s(s+A_1(p))}\, G_a(0,\ p) + \frac{1}{s+A_1(p)}\, t_{ai}(0,\ p), \\ t_w(s,\ p) &= \frac{b_3}{s+B_1(p)}\, t_a(s,\ p) + \frac{b_4}{s(s+B_1(p))}\, G_a(0,\ p) + \frac{1}{s+B_1(p)}\, t_{wi}(0,\ p). \end{aligned} \tag{1}$$

Then an inverse Laplace transform is performed with respect to the spatial coordinate for each equation separately and (1) is solved for the variables $t_a(H,\ p)$ and $t_w(H,\ p)$ to get the transfer functions as

$$W(p,\ H) = \frac{t_a(H,\ p)}{t_{ai}(0,\ p)} = \frac{e^{-A_1(p)H}}{1 - a_2 b_3 e^{-(B_1(p)+A_1(p))H}}. \tag{2}$$

This approach makes it comparatively simple and easy to derive the transfer functions, especially as regards analysis of the effects of parameters on equipment performance; the method has been tested via analytical calculations on the dynamic and static characteristics of an air cooler, and these were compared with measurements. The results were in good agreement. For example, the coefficient given by (2) in this method was $K_{t_{ai}} = 0.84$, while experiment gave $K_{t_{ai}} = 0.82$.

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